

# A Modal Superposition Based Method for Real-Time Structural Displacement and Force Reconstruction Using Sparse Sensor Data

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## Abstract

Reliable estimation of full-field structural displacements and internal forces from limited sensor data remains a key challenge in Structural Health Monitoring (SHM), particularly during strong dynamic events. This study proposes the Modal Structural Stress Mapping (MSSM) method, a real-time reconstruction technique for structural displacements and internal forces using sparse sensor measurements and modal superposition. Unlike traditional SHM approaches that detect damage through modal parameter shifts, MSSM provides direct physical response estimations that can support rapid damage localization and performance evaluation. The method has been implemented in a structural analysis software platform, integrated with an IoT-based SHM workflow, and validated through both numerical simulations and the instrumented Van Nuys hotel testbed. The method extends beyond seismic applications and is suitable for various structural typologies, including fatigue assessment under wind and vehicular loads. It provides a robust framework for evaluating long-term structural integrity and identifying potential damage in both structural and non-structural elements based on observed deformation patterns.

*Keywords:* Structural Health Monitoring, Modal Structural Stress Mapping (MSSM), Modal Superposition, Seismic Response Reconstruction, Digital Twin, Real-time Structural Monitoring, Finite Element Implementation

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## 1. Introduction

Structural health monitoring (SHM) has become a vital research area for assessing the integrity and performance of structures under operational and extreme loads. Recent reviews such as Karakostas et al. (1) have highlighted the growing importance of SHM methods in the seismic assessment of structures, particularly bridges, emphasizing the need for accurate full-field deformation estimation. Traditional SHM approaches often utilize vibration-based methods to detect damage and monitor the dynamic response of structures. These techniques typically rely on changes in global modal parameters—such as natural frequencies, damping ratios, and mode shapes—as indirect indicators of structural degradation or anomalies (2; 3). While such methods are effective for detecting global shifts in structural behavior, they are frequently insufficient for identifying damage at the component level. Many SHM systems, for instance, apply filters to sensor records to determine modal influence and structural displacements, which are then compared with predefined thresholds (15; 16). However, the results from these conventional approaches often offer limited insight into actual structural response under load and do not specifically provide information about the damage state of individual structural elements. This limitation has led to an increased demand for SHM methodologies that provide direct estimations of physical responses, such as displacements and internal forces, enabling more localized and actionable damage assessment (4).

Modal analysis, particularly Operational Modal Analysis (OMA), allows the identification of structural mode shapes using vibration data, without requiring knowledge of input forces. OMA has been widely adopted in structural engineering due to its non-invasive nature and suitability for in-service structures (5; 6). However, while OMA can identify change in mode shapes, it does not provide direct information about internal forces or displacements under seismic or wind loads (7).

Guyan's seminal 1965 paper (8) presented a foundational method for reducing the order of stiffness and mass matrices within engineering systems. This technique, commonly referred to as "Guyan reduction," effectively simplifies the computational model by systematically condensing out non-essential degrees of freedom (DOFs). This methodology has proven indispensable for facilitating the computationally efficient dynamic analysis of large and intricate structural assemblies.

Several studies have explored hybrid models that combine sensor data

with finite element models (FEMs), often within digital twin frameworks or through model updating techniques (9). These approaches aim to enhance predictive accuracy by coupling physical models with real-time monitoring data, yet they frequently rely on complex optimization or inverse problem formulations, which can be computationally intensive and sensitive to modeling uncertainties.

Various computational approaches have also been proposed to estimate full-field structural responses from limited nodal displacement data. Classical interpolation methods such as polynomial fitting, splines, or Kriging aim to approximate displacements at unmeasured locations, but they generally do not consider the structural stiffness or dynamic properties. Model updating techniques and inverse finite element methods attempt to adjust model parameters to minimize the discrepancy between observed and simulated responses, yet they are often computationally expensive and suffer from non-uniqueness. State estimation approaches like Kalman filters or nonlinear observers offer real-time capabilities but are highly sensitive to model assumptions and noise. Modal expansion techniques reconstruct structural fields based on measured modal coordinates and known mode shapes, though they may require extensive modal data and often assume linear behavior.

Roohi et al.(13) proposed a seismic monitoring framework that employs a nonlinear model-based state observer to estimate the seismic response of instrumented buildings, using dissipated energy as a feature for damage detection and localization. This framework integrates a nonlinear finite element model with global acceleration measurements to estimate the time history of the seismic response. In parallel, unsupervised learning-based anomaly detection methods have been proposed to identify and localize damage directly from sensor data without requiring labeled damage scenarios (14). While these approaches are promising in terms of automation and generalizability, they often lack a direct link to physical quantities such as displacement or internal force, limiting their interpretability in engineering practice.

In this article, instead of vibration data-based methods that have been used so far, a method based on the combination of mode shapes will be introduced. This method, which is based on the superposition of structural responses under earthquake and wind effects with certain coefficients of natural vibration modes, can obtain all deformations of the structure by calculating modal participation factors throughout the strong motion (17; 18). The method is named “Modal Structural Stress Mapping” (MSSM) as it enables the estimation of deformations and stresses across the entire structure using

a combination of mode shapes. In contrast to these existing approaches, the MSSM method proposed in this study offers a direct and efficient technique to reconstruct global structural displacements and internal forces using modal superposition from obtained sensor data. Because MSSM provides element-level estimations of internal forces and deformations, it can also reveal hidden damage in structural components that may not be externally visible, thereby enabling more reliable and interpretable condition assessments.

The proposed MSSM method provides a computationally efficient framework that leverages known mode shapes and sparse sensor data without reducing the system degrees of freedom. Unlike traditional reduction or interpolation schemes, MSSM retains the full dynamic characteristics of the structure and enables the real-time estimation of both displacement and internal force fields.

The method aligns with current SHM trends that emphasize the integration of digital twins, sensor networks, and real-time decision-making tools for structural performance assessment. To this end, recent research has focused heavily on enabling real-time analysis capabilities in digital twins—through techniques such as reduced-order modelling and data-driven acceleration—to support on-the-fly structural evaluation for safety and resilience (12). Model updating techniques can also be employed prior to the application of MSSM to enhance the fidelity of the structural model, enabling more accurate modal representations and improving the reliability of the reconstructed responses.

In addition to seismic analysis, the MSSM method is also suitable for deformation and fatigue analyses under wind and vehicle loads. A particularly advantageous application of the MSSM method involves identifying damage in non-structural elements by analyzing the deformations of structural components, especially in cases where the structure exhibits linear behavior and remains undamaged during seismic events. Since the deformation field of the structure is precisely reconstructed, the method allows for accurate prediction of potential damage and associated costs in non-structural components, particularly those sensitive to interstory drift or excessive displacement. In particular, following weak seismic events where no apparent structural damage is observed, the internal force estimates obtained via MSSM can be compared with element capacities. If the computed demands are significantly lower, rapid decisions can be made regarding continued occupancy and operational continuity.

Main contributions of this study can be summarized as follows:

- Proposes the "Modal Structural Stress Mapping" (MSSM) method, which enables full-field displacement estimation at all structural joints using limited sensor data.
- Allows mass definition at arbitrary nodes and degrees of freedom without applying any model reduction, thereby preserving the complete dynamic characteristics of the structure—unlike conventional Guyan reduction.
- Enables the identification of both structural and non-structural damage through deformation-based assessment, and provides information on hidden damage in structural elements, allowing for targeted inspections.
- Allows for quick decisions regarding the continued use of the structure in certain scenarios.
- Supports the analysis of wind, traffic, and other vibration-induced loads in addition to seismic excitations, as long as the response can be expressed through mode superposition.
- Integrates the proposed method into the structPlus finite element software, which automates analysis, reporting, and performance evaluation.
- Includes the generation of Modal Assurance Criterion (MAC) matrices to assess and optimize sensor locations and monitoring directions.
- Facilitates fatigue monitoring by tracking the number of tension–compression cycles experienced by structural elements over time.

## 2. Proposed Method

In the field of structural dynamics, the modal participation factor (often denoted as  $\Gamma$ ) is a crucial parameter that quantifies the contribution of each vibration mode to the overall response of a structure. It essentially indicates how much a specific mode participates in the global motion of the structure when subjected to dynamic loads, such as earthquakes or wind. When a structure vibrates, it does so in a combination of different patterns called modes. The modal participation factor for a given mode is a scalar value

that reflects the degree to which that mode is excited by a particular dynamic load.

Modal analysis methods are frequently employed to investigate the behavior of structures under dynamic loads such as wind or earthquakes. In these analyses, the motion of the structure over time can be determined by utilizing mode shapes in conjunction with time-varying modal participation factors. Modal participation factors serve as a dynamic measure indicating the contribution of each mode to the overall response of the structure. These factors can vary depending on the load applied to the structure and its change over time. Therefore, in order to accurately model the dynamic response of the structure, it is important to consider the change in modal participation factors over time.

The motion of a structure at any given time can be expressed by using the natural vibration modes of the structure together with their modal participation ratios at that time as seen in Equation 1.

$$\mathbf{d} = \Phi \Gamma \quad (1)$$

$\mathbf{d}$ : Displacement vector for structural freedoms

$\Phi$ : Modal shape matrix for structural freedoms

$\Gamma$ : Vector containing modal participation factors

However, it may not be possible to determine all mode shapes of the structure. In this case, using a sufficient number of mode shapes in the same approach is approved by regulations (ASCE7(19) section 12.9.1, TBDY2018(20) section 4.8.1.2) and many scientific sources. Equation 2 provides the relationship that links the modal participation factors to the joint displacements

$$[d]_{n \times 1} = [\Phi]_{n \times m} [\Gamma]_{m \times 1} \quad (2)$$

n: Number of structural freedoms

m: Number of structural mode shapes obtained

During dynamic events like earthquakes or wind loads, structural sensors capture motion data at specific points. While the motion of all structural elements cannot be directly derived from these recordings, the contribution of individual mode shapes can be determined by considering the superposition of modes. At each time step, the modal participation factors corresponding to the selected mode shapes—equal in number to the sensor channels—are computed by solving a system of linear equations. Since all the observed

channels correspond to a degree of freedom in the structure, this set of equations will have as many unknowns as the number of channels. Figure 1 illustrates the relationship between mode shapes and sensors.

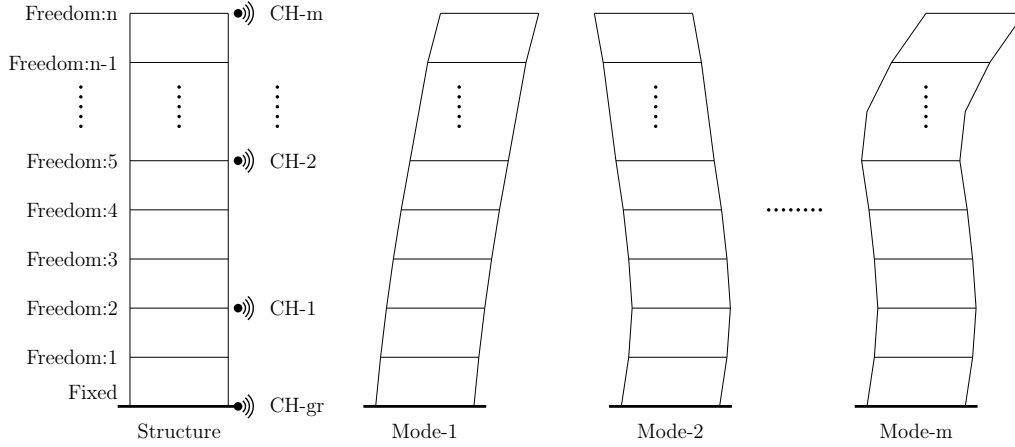


Figure 1: Freedoms of structure, sensors and corresponding modes

The relationship connecting modal participation factors to sensor point displacements is given in Equation 3.

$$[d]_{m \times 1}^{CH} = [\Phi]_{m \times m}^{CH} [\Gamma]_{m \times 1} \quad (3)$$

$[d]_{m \times 1}^{CH}$ :  $m \times 1$  vector indicating displacement data obtained from sensors (removed ground motion)

$[\Phi]_{m \times m}^{CH}$ :  $m \times m$  matrix from mode shapes for freedoms containing sensors

$[\Gamma]_{m \times 1}$ :  $m \times 1$  vector for modal participation factors

The illustration was shown on a simple shear frame, but the procedure introduced can be applied to any problem that can be defined in terms of mode shapes. After computing the modal participation factors for a given time step, the global structural displacements, internal member forces, and stresses can be reconstructed through mode shape superposition.

After obtaining displacements and internal forces due to seismic excitation, the additional effects of vertical loads can be incorporated to assess member performance. Using the elastic deformations of the structural elements, the damage that occurs under the deformation in the elements can be determined, as well as the demand/capacity ratios of the elements can be

calculated from the internal forces obtained. Moreover, full-field deformation estimates enable the assessment of potential damage in non-structural components.

The modal structural stress mapping (MSSM) procedure consists of the following steps:

1. A digital twin must be setup to use in analysis. The digital twin can also be updated by using sensor data.
2. Appropriate mode shapes for the problem to be solved are selected. At this stage, sensor locations can also be positioned to best represent the mode shapes. The suitability of sensor locations can be checked from MAC tables.
3.  $[\Phi]_{m \times m}^{CH}$  matrix will be calculated. At this step, LU Decomposition is applicable.
4. After strong ground motion occur,  $[d]_{m \times 1}^{CH}$  vector is prepared for each time step of ground motion. ( $[d]_{m \times k}^{CH}$  for k steps)
5. The equation system  $[d]_{m \times 1}^{CH} = [\Phi]_{m \times m}^{CH} [\Gamma]_{m \times 1}$  will be solved for each time step to find  $[\Gamma]_{m \times 1}$
6.  $[\Gamma]_{m \times 1}$  will be used in equation  $[d]_{n \times 1} = [\Phi]_{n \times m} [\Gamma]_{m \times 1}$  to obtain structural displacements for each time step.
7. The internal forces and stresses can be calculated for each time step.
8. The effect of vertical loads will be added to ground motion results to find overall forces and stresses
9. Using the MSSM procedure, structural assessments such as interstory drift, stiffness irregularities, and torsional response can be systematically evaluated.
10. The member seismic performance can be calculated by using deformations, demand/capacity ratios of the elements can be calculated from the internal forces obtained.
11. Non-structural member damages may also be determined using deflections.

Since the system of equations is linear, LU decomposition can be used to enhance computational efficiency(21). The flow diagram of the analysis steps explained above is given in Figure 2 and 3.

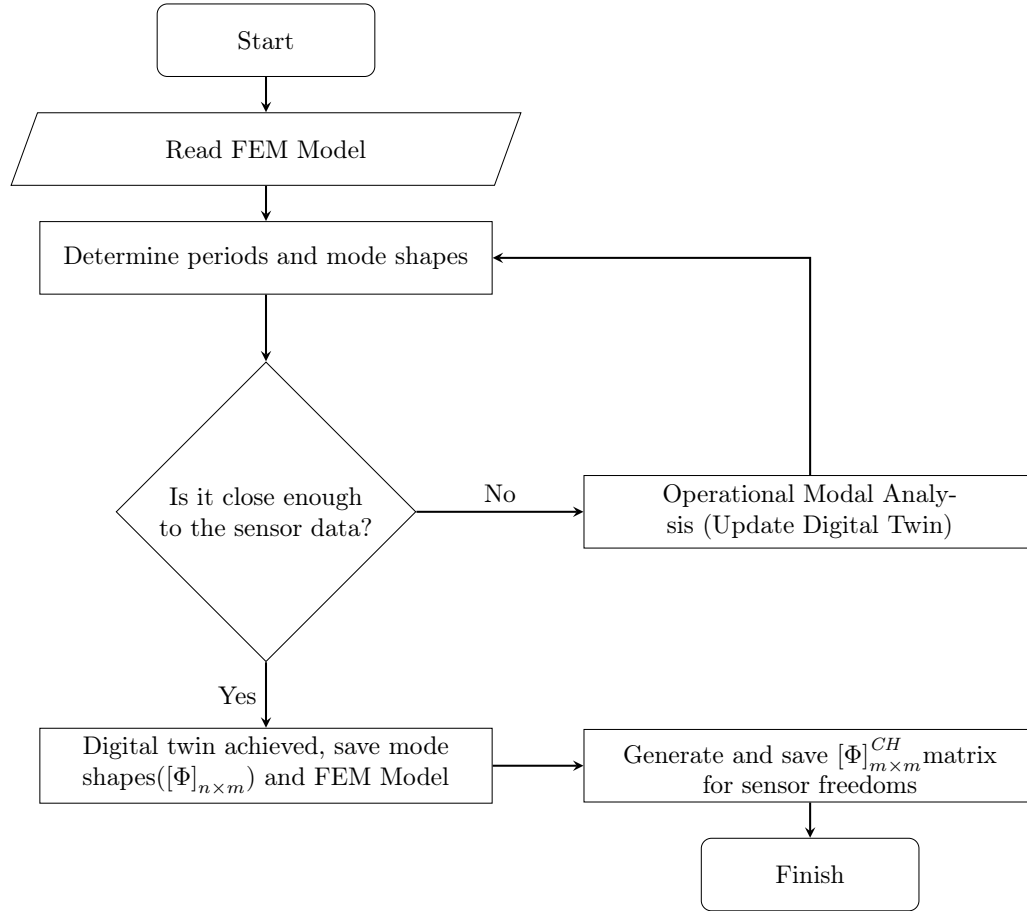


Figure 2: Flowchart for determining mode shapes and equation sets before seismic or wind activity

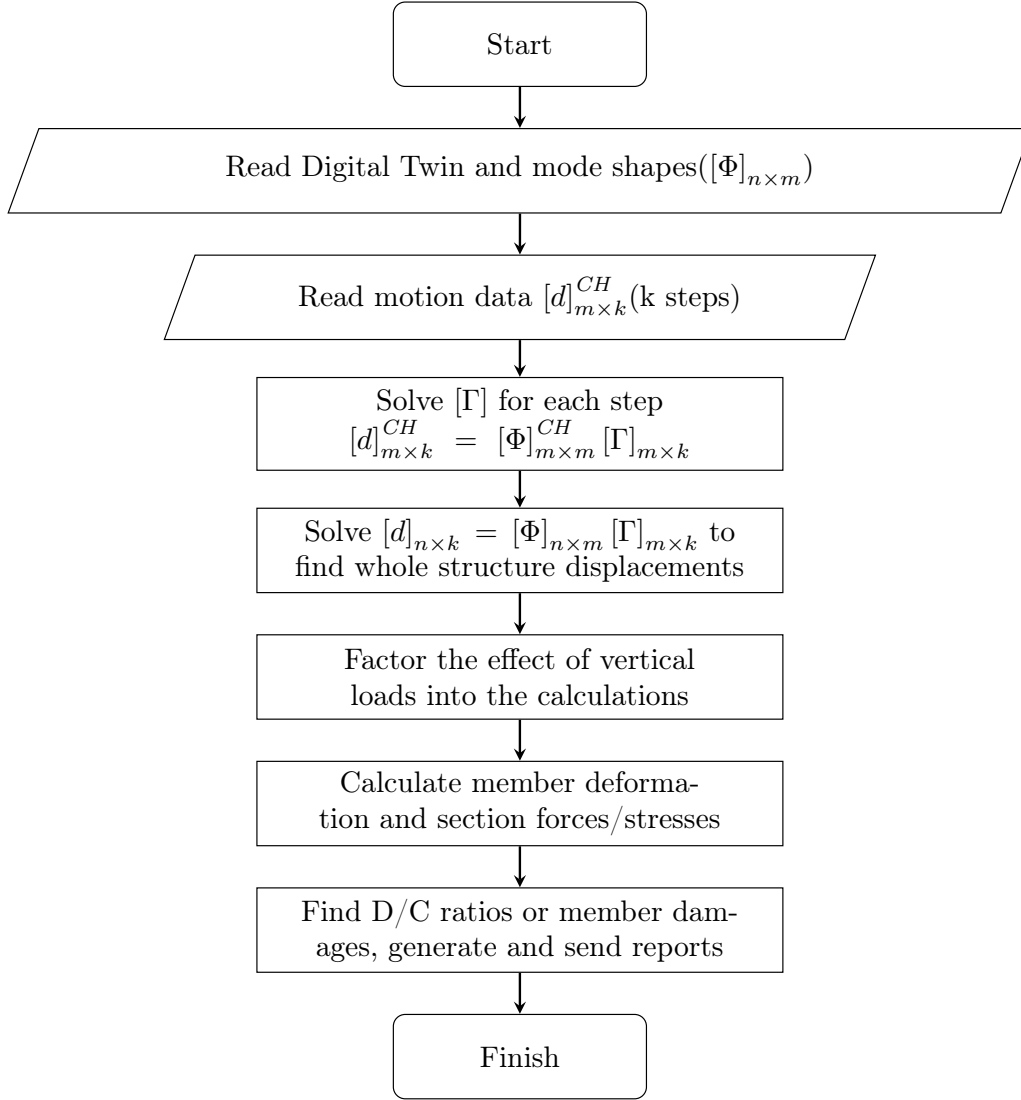


Figure 3: Flowchart for executing strong ground motion or wind forces

### 3. Illustrative Case Study

To better illustrate the MSSM procedure and to allow for its independent verification, the method was applied to a six-story, two-dimensional frame. The frame consists of columns with dimensions of 60x60cm and beams with dimensions of 40x60cm. Concrete with an elasticity modulus of 33000MPa was used as the material. A mass of 5000kg was assigned to each node of

the frame, which has a bay width of 5m and a story height of 3m. Dynamic analysis was performed using Direct Integration, and damping parameters were determined using the first two natural vibration periods of the structure. The sensors are located in 3rd and 6th stories. Figure 4 shows the simple frame used in analysis.

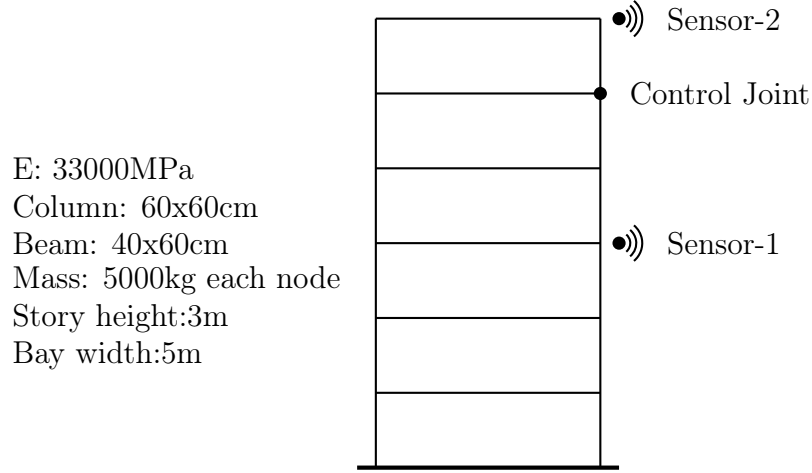


Figure 4: Simple frame properties, sensor locations and control joint

Table 1 shows structure periods and first two mode shapes.

Table 1: First two periods and mode shapes for given example

|             | Mode 1 | Mode 2  | Joint Information |
|-------------|--------|---------|-------------------|
| T(sec)      | 0.3390 | 0.1035  |                   |
| Story 6(mm) | 0.1884 | -0.1722 | Sensor 2          |
| Story 5(mm) | 0.1696 | -0.0536 | Control Joint     |
| Story 4(mm) | 0.1416 | 0.0822  |                   |
| Story 3(mm) | 0.1052 | 0.1692  | Sensor 1          |
| Story 2(mm) | 0.0636 | 0.1625  |                   |
| Story 1(mm) | 0.0231 | 0.0754  |                   |
| Base        | 0.0000 | 0.0000  | Fixed Joint       |

According to table,  $[\Phi]_{m \times m}^{CH}$  matrix is prepared as seen in Equation 4 independent from earthquake records:

$$[\Phi]_{2 \times 2}^{CH} = \begin{bmatrix} 0.10521 & 0.169162 \\ 0.1884 & -0.17218 \end{bmatrix} \quad (4)$$

Linear direct integration analyses was performed on model using 1999 Kocaeli earthquake (Peer Data: RSN1176\_KOCAELI\_YPT060(22)) without any scaling. The record data for two defined sensors in the structure were obtained from the results of Time History Analysis. These outputs were then used as input for the MSSM procedure, and the horizontal displacement of the 5th floor joint was calculated using MSSM and compared with the Time History Analysis results.

Throughout the analysis, the minimum and maximum displacements occurred at 10.535 seconds and 16.525 seconds, respectively. At the mentioned instant of the analysis (t=10.535 sec), the displacement on sensor 1 and sensor 2 is -11.968mm and -20.931mm respectively. Hence, the calculation of the modal participation factors and the formation of the system introduced in Equation 3 are explicitly shown below:

$$\begin{bmatrix} 0.10521 & 0.169162 \\ 0.1884 & -0.17218 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \begin{bmatrix} -11.968 \\ -20.931 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \begin{bmatrix} -112.061 \\ -1.04986 \end{bmatrix} \quad (6)$$

After obtaining the participation factors, joint displacements can be calculated using mode superposition. As previously mentioned in Equation 2, to calculate structural displacement, the displacements of two modes are added by multiplying their participation ratios, as explicitly calculated in Equation 7.

$$\begin{bmatrix} -20.931 \\ -18.959 \\ -15.955 \\ -11.968 \\ -7.3038 \\ -2.6728 \end{bmatrix} = \begin{bmatrix} 0.1884 & -0.17218 \\ 0.16969 & -0.0536 \\ 0.14161 & 0.082227 \\ 0.10521 & 0.169162 \\ 0.06365 & 0.162538 \\ 0.02315 & 0.075406 \end{bmatrix} \begin{bmatrix} -112.061 \\ -1.04986 \end{bmatrix} \quad (7)$$

All quantities related to the structural elements, such as element deformations, section forces, and plate stresses, can be obtained using the mode superposition method under the assumption of linear behavior. First and second story moments are compared in Figure 5.

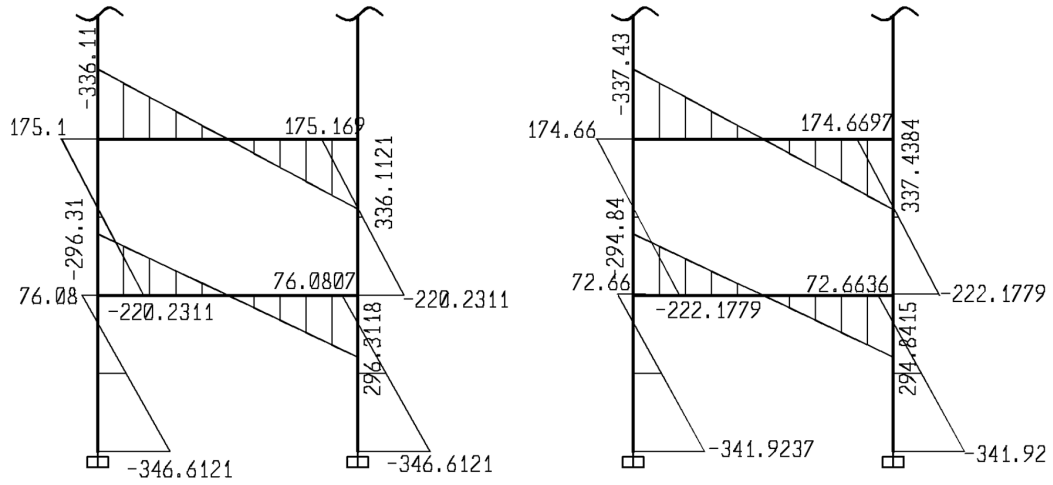


Figure 5: First two story frame moments obtained from Time History(left) and MSSM(right)

To observe the consistency of the proposed method, the horizontal displacement of the 5th floor node calculated by MSSM was compared with the Time History Analysis result throughout the analysis.

## Displacement of 5<sup>th</sup> story

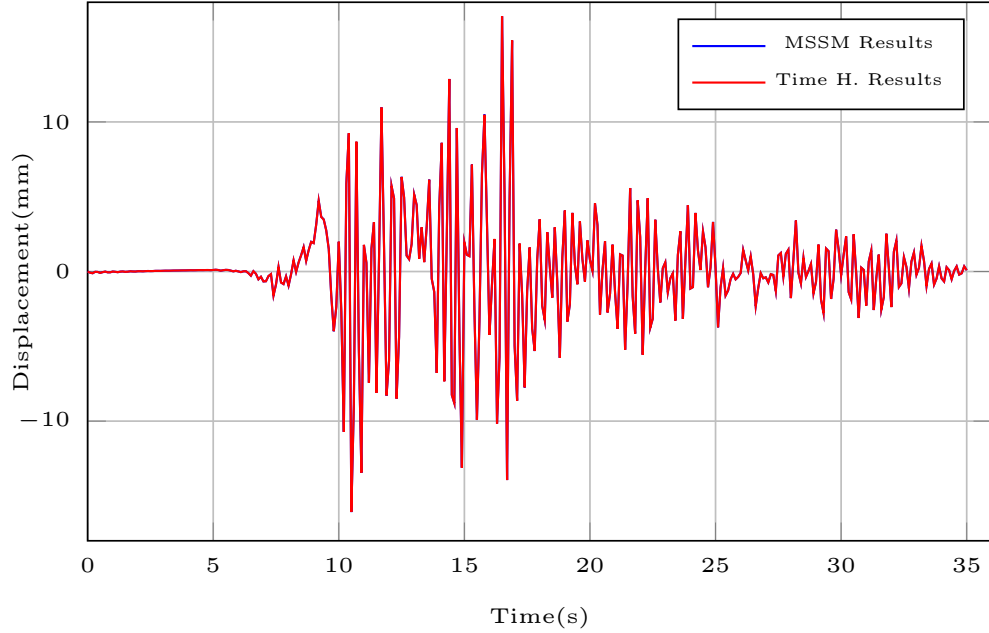


Figure 6: Direct Time History and MSSM comparison of 5<sup>th</sup> story displacements

As seen in Figure 6, the horizontal displacements obtained with the MSSM method are so close to those obtained with Time History Analysis that they progress along the same point throughout the entire graph. Furthermore, the key quantities at the analysis steps where maximum and minimum displacements occurred are provided in Table 2, encompassing the five preceding and five subsequent steps as well.

The maximum horizontal displacement difference for this model is 0.12%, which is very close to Time History Analysis results. Total modal participating mass ratio is 92% on the model, so the first two modes can be considered sufficient to represent the structural behavior.

#### 4. Software Implementation with structPlus

The explained MSSM procedure has been adapted to the finite element software called structPlus. structPlus was developed within the scope of the author's Ph.D. thesis, "Nonlinear Analysis of Reinforced Concrete Structures

Table 2: Comparing horizontal displacements(mm) for steps with minimum and maximum displacements for node at story 5

| Step | Time   | dX-Tm. Hst. | dX-MSSM | Diff. % | $\Gamma$ : Mod 1 | $\Gamma$ : Mod 2 |      |
|------|--------|-------------|---------|---------|------------------|------------------|------|
| 2103 | 10.510 | -17.53      | -17.54  | 0.08834 | -103.65          | -0.849           |      |
| 2104 | 10.515 | -18.07      | -18.09  | 0.09131 | -106.90          | -0.961           |      |
| 2105 | 10.520 | -18.49      | -18.50  | 0.08436 | -109.37          | -1.042           |      |
| 2106 | 10.525 | -18.77      | -18.79  | 0.07001 | -111.06          | -1.086           |      |
| 2107 | 10.530 | -18.93      | -18.94  | 0.05688 | -111.96          | -1.088           |      |
| 2108 | 10.535 | -18.95      | -18.96  | 0.04547 | -112.06          | -1.050           | Min. |
| 2109 | 10.540 | -18.84      | -18.85  | 0.04887 | -111.37          | -0.975           |      |
| 2110 | 10.545 | -18.59      | -18.60  | 0.05254 | -109.90          | -0.868           |      |
| 2111 | 10.550 | -18.22      | -18.23  | 0.06368 | -107.65          | -0.738           |      |
| 2112 | 10.555 | -17.72      | -17.73  | 0.07293 | -104.66          | -0.597           |      |
| 2113 | 10.560 | -17.09      | -17.11  | 0.07221 | -100.96          | -0.460           |      |
| .... |        |             |         |         |                  |                  |      |
| 3301 | 16.500 | 17.06       | 17.08   | 0.11669 | 100.89           | 0.829            |      |
| 3302 | 16.505 | 17.62       | 17.64   | 0.09932 | 104.19           | 0.836            |      |
| 3303 | 16.510 | 18.06       | 18.08   | 0.08418 | 106.78           | 0.839            |      |
| 3304 | 16.515 | 18.37       | 18.39   | 0.06839 | 108.62           | 0.830            |      |
| 3305 | 16.520 | 18.56       | 18.57   | 0.05336 | 109.67           | 0.801            |      |
| 3306 | 16.525 | 18.60       | 18.61   | 0.04256 | 109.92           | 0.749            | Max. |
| 3307 | 16.530 | 18.51       | 18.52   | 0.03029 | 109.35           | 0.672            |      |
| 3308 | 16.535 | 18.28       | 18.29   | 0.02921 | 107.96           | 0.572            |      |
| 3309 | 16.540 | 17.92       | 17.92   | 0.02752 | 105.77           | 0.457            |      |
| 3310 | 16.545 | 17.42       | 17.42   | 0.02684 | 102.79           | 0.334            |      |
| 3311 | 16.550 | 16.79       | 16.80   | 0.03279 | 99.04            | 0.213            |      |

on a Two-Parameter Foundation and a Visual-Based Computer Software"(23) and has been used for many years in numerous engineering applications. Joints, frames, shell or plates, node springs and shell springs can be defined in program. A screenshot showing the frame forces and shell stresses of a suspension bridge is shown in Figure 7. The program incorporates numerous static analysis techniques, including linear analysis, dynamic analysis with eigen or Ritz vectors, time history analysis, and soil-structure interaction analysis. It is also extensively utilized for post-linear analysis operations such as reinforced concrete or steel element design, and structural performance assessment by determining element damage. For nonlinear calculations, the program generates Etabs(24) or OpenSees(25) input files; after these are run in the respective software, the results are imported back into structPlus to continue the design process. Currently, nonlinear analyses con-

cerning material or geometry are not available directly within the program.

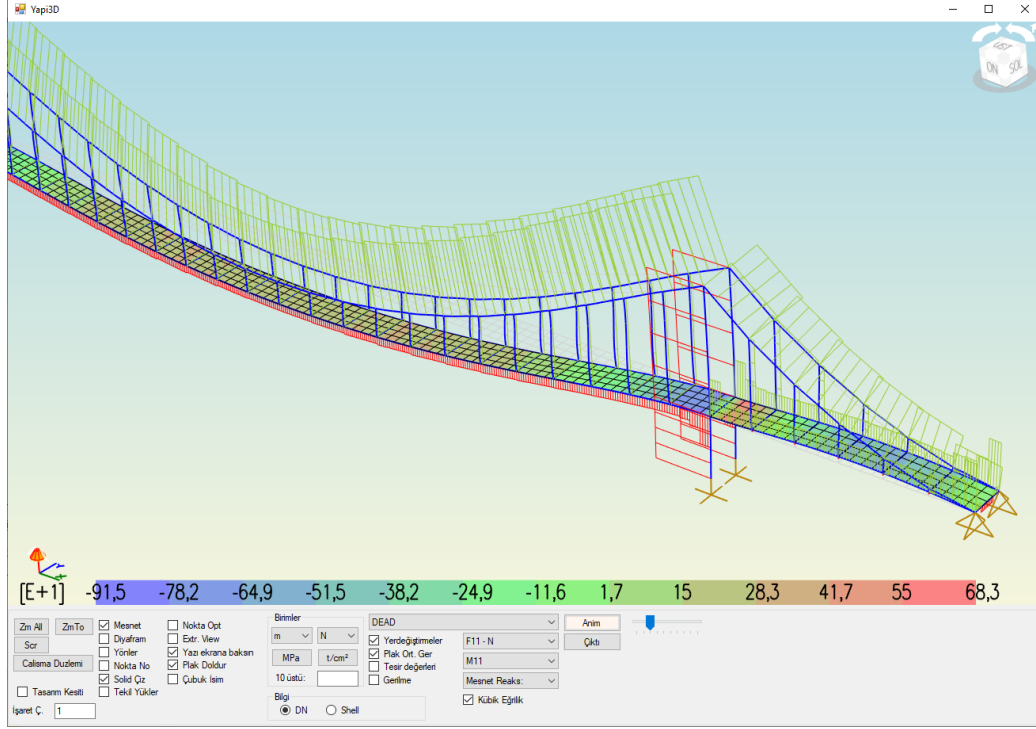


Figure 7: Frame forces and plate stresses are shown on the deformed shape of a suspension bridge.

structPlus is a stand-alone software and it can be downloaded from <https://web.itu.edu.tr/~avcioglu/>. The software is not open-source, but it is tightly integrated with CSI programs such as Sap2000 and Etabs, as well as with the OpenSees platform. A wide range of validation studies have been conducted by solving structural models using structPlus, Sap2000, Etabs, and OpenSees, and comparing their results. For SHM applications utilizing the MSSM method, the digital twin is typically prepared by developing the finite element model within Etabs. The Etabs model file is then imported into structPlus, where the consistency of computed mode shapes and structural periods between the two platforms is verified. After this verification step, the MSSM procedures are carried out within structPlus.

Over the last year, numerous enhancements related to structural health monitoring have been integrated into the structPlus software. These additions enable the examination of the structure before a seismic event, the

automatic analysis of data prepared on the server where the software is installed after a seismic event, and the submission of reports via its integrated IoT platform. The specific features added to the software for these operations include:

- Definition of sensor locations on joints and monitoring directions as degrees of freedoms.
- Generation of MAC matrices based on sensor locations and monitoring directions.
- Determination of mode shapes corresponding to natural vibrations using the eigenvector method.
- Formation of  $[\Phi]_{n \times m}$  matrix that connect the channels to all degrees of freedom and  $[\Phi]_{m \times m}^{CH}$  matrix to find modal participation factors for each time step of seismic record.
- Solution of  $[\Phi]_{m \times m}^{CH}$  matrix via LU decomposition.
- Automatic execution of the software on the server where it is installed by the integrated IoT platform after an earthquake, allowing the relevant digital twin and the acquired record to be loaded.
- Performing analyses on the server to determine displacements and element damages.
- Generation of report in PDF file format, detailed tables in CSV file format, and drawings showing damage on floor plans in DXF file format.
- Sharing these files with building directors through the IoT platform.

The sensor definition window in the structPlus software is used to define sensor locations on the building's digital twin. Figure 8 shows sensor definition window. Through the MAC matrices(26) obtained by considering the joints where the sensors are located, the optimization of sensor locations can also be done in this menu. Table 3 shows MAC matrices outputs from structPlus software.

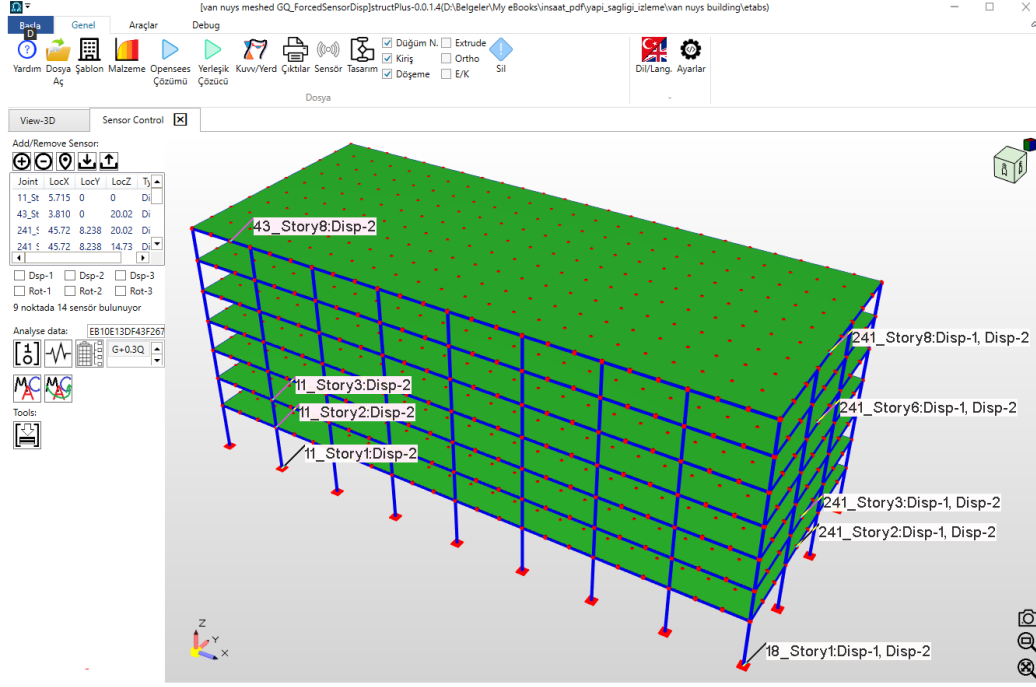


Figure 8: Sensor window

Table 3: MAC Table for Van Nuys building

| T:           | <b>2.473</b> | <b>1.166</b> | <b>0.842</b> | <b>0.782</b> | <b>0.438</b> | <b>0.369</b> | <b>0.289</b> | <b>0.265</b> | <b>0.206</b> | <b>0.141</b> | <b>0.088</b> |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <b>2.473</b> | 1            | 0.018        | 0.091        | 0.080        | 0.453        | 0            | 0.025        | 0            | 0.011        | 0.012        | 0            |
| <b>1.166</b> | 0.018        | 1            | 0.045        | 0.014        | 0.011        | 0.015        | 0.027        | 0.003        | 0.162        | 0.002        | 0.297        |
| <b>0.842</b> | 0.091        | 0.045        | 1            | 0.188        | 0.003        | 0            | 0.082        | 0.018        | 0.014        | 0.108        | 0.003        |
| <b>0.782</b> | 0.080        | 0.014        | 0.188        | 1            | 0.042        | 0.001        | 0.193        | 0.042        | 0            | 0.010        | 0.014        |
| <b>0.438</b> | 0.453        | 0.011        | 0.003        | 0.042        | 1            | 0.005        | 0            | 0.017        | 0.008        | 0.060        | 0.017        |
| <b>0.369</b> | 0            | 0.015        | 0            | 0.001        | 0.005        | 1            | 0.002        | 0.051        | 0.066        | 0.076        | 0.253        |
| <b>0.289</b> | 0.025        | 0.027        | 0.082        | 0.193        | 0            | 0.002        | 1            | 0.037        | 0.003        | 0.002        | 0.007        |
| <b>0.265</b> | 0            | 0.003        | 0.018        | 0.042        | 0.017        | 0.051        | 0.037        | 1            | 0.014        | 0.017        | 0.554        |
| <b>0.206</b> | 0.011        | 0.162        | 0.014        | 0            | 0.008        | 0.066        | 0.003        | 0.014        | 1            | 0.116        | 0.013        |
| <b>0.141</b> | 0.012        | 0.002        | 0.108        | 0.010        | 0.060        | 0.076        | 0.002        | 0.017        | 0.116        | 1            | 0            |
| <b>0.088</b> | 0            | 0.297        | 0.003        | 0.014        | 0.017        | 0.253        | 0.007        | 0.554        | 0.013        | 0            | 1            |

The values in the MAC tables range between 0 and 1, indicating the degree of correlation between different mode shapes at the sensor locations. The diagonal values, which are always equal to 1, represent the correlation of each mode with itself. Ideally, all off-diagonal values should be zero, indicating perfect modal orthogonality at the sensor locations. Although this ideal condition is rarely achieved in practice, the closer the off-diagonal

values are to zero, the better the sensor placement is considered to be.

Sensor definition window can prepare and save modal analysis results to use later. This window can also read pre-recorded sensor records and start MSSM procedure.

MSSM procedure can be started manually by reading the digital twin and records. Also structPlus can automatically start procedures to obtain results on servers. This feature enables automated report generation and delivery to clients via email.

Displacement and internal force results can be examined through the software's window, as illustrated in Figure 8.

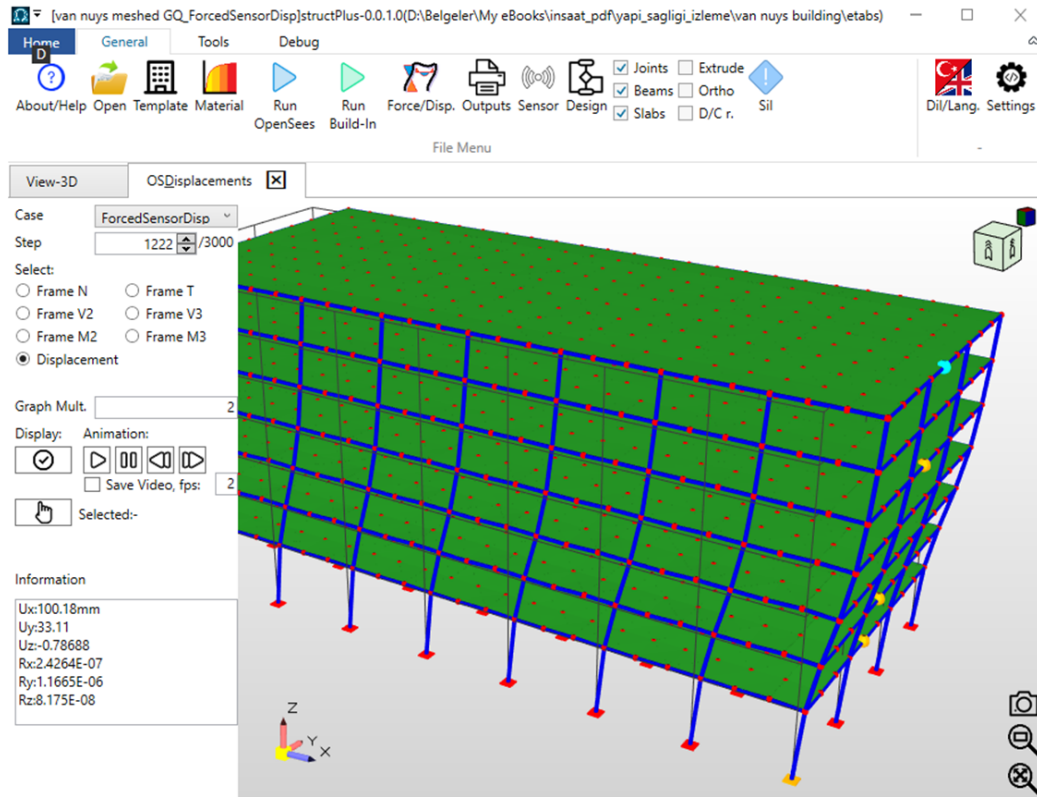


Figure 9: Displacement results window

## 5. Validation On Van Nuys Hotel Testbed

The proposed method is validated using data from the Van Nuys hotel testbed, a seven-story reinforced concrete building (Figure 10) instru-

mented by the California Strong Motion Instrumentation Program(27). Initial instrumentation of the structure included three independent triaxial accelerographs installed prior to the 1971 San Fernando earthquake. Ahead of the 1987 Whittier-Narrows earthquake, the system was upgraded to a more comprehensive 16-channel configuration incorporating uniaxial, biaxial, and triaxial sensors. The building has 14 channels that record horizontal displacements(28). Three of these channels are at the base level, and the remaining 11 are independent of the base. Accordingly, 11 mode shapes should be used to examine the Van Nuys building with the MSSM method. Sensor placement and the channels monitored by the sensors are shown in Figure 12.



Figure 10: View of Van Nuys Seven Story Hotel from North-East(source: Gicev et al. (29)).

The mass participation ratios of the first 11 modes are Mux: 99.7%, Muy: 99.49%, and Mrz: 99.12%, respectively. These ratios are quite sufficient for examining the dynamic behavior of the structure. If we remove one of the channels in the structure from the calculations, the mass participation ratios decrease to Mux: 99.7%, Muy: 98.52%, and Mrz: 99.12%. As can be seen from this, there is only a very small loss in the mass participation in the UY direction.

To test the method, the CHAN06 data, which monitors the Y direction on the second floor (labeled as Story3), was excluded, and the displacements obtained with the remaining records were compared with the CHAN06 data. During this trial, since 10 modes were used instead of 11 due to the removed channel, the modal participation decreased. Therefore, it is obvious that the same displacements will not be obtained. Also, since each channel does not follow a specific mode, the change in the modal participation ratio used in the calculations is not entirely clear. Despite all this, the displacement of the removed channel was obtained very closely as seen in Figure 11.

### Reconstructing CHAN06 Data

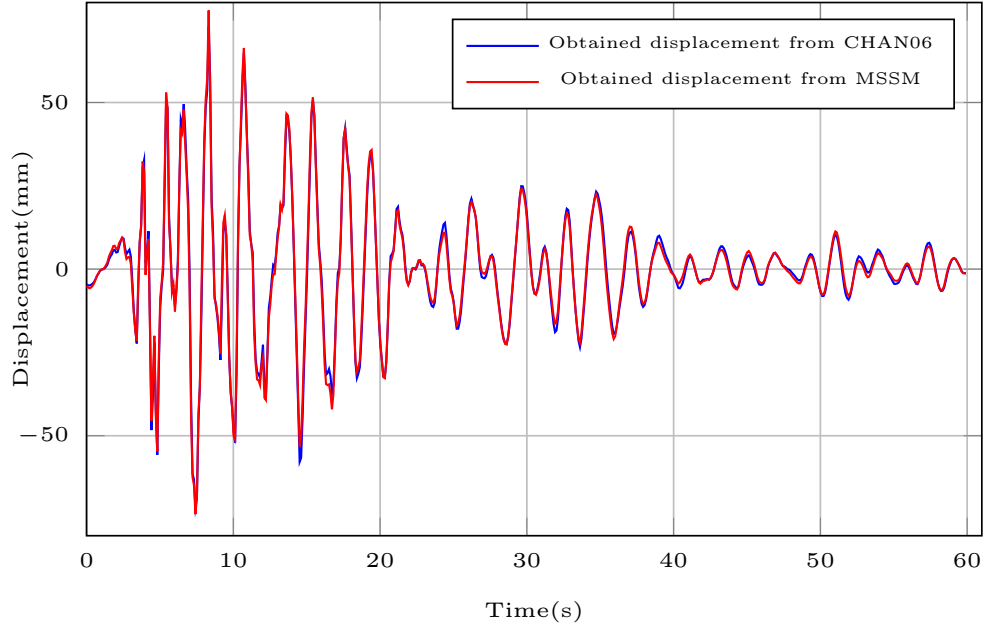


Figure 11: Comparison of sensor data and reconstructed displacement using MSSM

The maximum displacement(+Y) from Channel 6 is 74.4 mm, and the displacement calculated with the MSSM method is 77.875 mm, corresponding to a difference of 4.67%. Similarly, the minimum displacement(-Y) is 73.06 mm, and the displacement calculated with the MSSM method is 73.75 mm, corresponding to a difference of 0.9%. While these differences are sufficiently small for structural analysis purposes, they may have been further

influenced by the limitations of the instruments—originally developed using 1980s technology—and the filtering applied to the recorded data. In addition, the exclusion of one vibration mode in the validation process may also contribute to the observed deviation from the recorded sensor response.

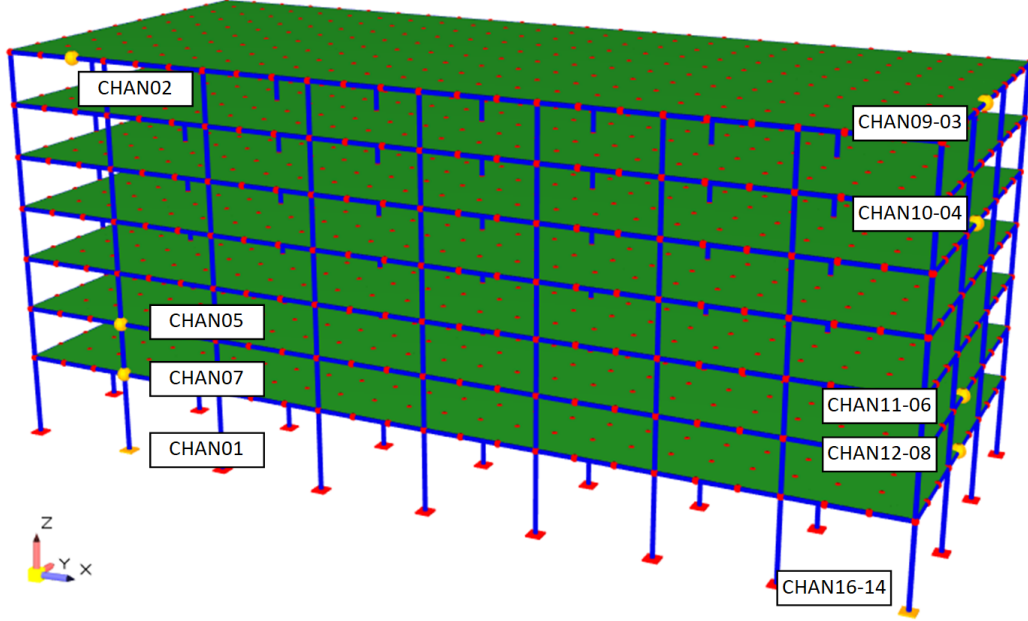


Figure 12: Sensor locations and lateral direction channel definitions (X and Y directions respectively)

## 6. Limitations

The method discussed in this paper operates on the principle of mode superposition and therefore relies on the assumption of linear elastic behavior. Under a real earthquake, as the building begins to be stressed, some structural members will start to exhibit nonlinear behavior. Since the initial damage will not significantly alter the structural mode shapes, the system will still be able to identify these elements. However, after structure's mode shapes changes due to strong motion, the system can detect significant damage in the structure, but unforeseen elements may also appear in the damage distribution. As the use of such a heavily stressed structure would not be appropriate anyway, repairs would be necessary after this stage. Even in

such cases, the resulting damage distributions can provide valuable guidance for post-event site inspections.

In cases where buildings are not excessively stressed, the MSSM system will indicate that the structure is operational. Also information about non-structural member damages will also be obtained through this system.

As plastic deformations begin to significantly alter the dynamic characteristics of the structure, the linear assumption underlying the MSSM method becomes less valid. However, to account for nonlinear behavior beyond the onset of yielding, a revised modal analysis can be performed at selected stages of the seismic event. By updating the mode shapes to reflect the damaged state of the structure, new  $[\Phi]_{n \times m}$  and  $[\Phi]_{m \times m}^{CH}$  matrices can be generated and used in the MSSM procedure. This approach allows the method to approximate nonlinear behavior by incorporating updated dynamic properties at different stages of the response.

The implementation of this approach is planned as part of future work.

## 7. Conclusion

This study presents the Modal Structural Stress Mapping method, a novel approach for reconstructing structural deformations and internal forces using a limited number of mode shapes and sensor data. Unlike traditional vibration-based structural health monitoring (SHM) techniques, which primarily detect changes in modal parameters, MSSM enables direct calculation of global displacements and stresses by solving time-dependent modal participation factors. The method utilizes a linear system of equations based on modal superposition, allowing rapid and efficient computation even during real-time monitoring scenarios.

Validation using both a simplified structural model and the Van Nuys hotel testbed demonstrates that MSSM provides results with high accuracy, closely matching those obtained from full Time History Analyses. The integration of MSSM into the structPlus software further enhances its practical applicability, offering automated workflows for structural analysis, monitoring, and performance evaluation.

The examples presented in this study focus on building-type frame structures due to their simplicity and the availability of test data, which facilitate clarity and validation. However, the application of the proposed method is not limited to buildings. The MSSM framework can be extended to any

structural system that can be described through modal behavior, including bridges, towers, and wind turbines.

By providing a direct link between recorded sensor data and full-field structural response, MSSM significantly advances the capabilities of SHM systems. It allows not only the assessment of structural performance under seismic and wind loads but also the evaluation of element-level demands, damage states, and non-structural component impacts. As such, MSSM emerges as a robust and scalable framework for enhancing the resilience of buildings and other structural systems, while enabling real-time damage assessment.

## **8. AI Use Disclosure**

The manuscript was solely authored by the submitting author. OpenAI's ChatGPT (version GPT-4) and Google's Gemini(version Flash-2) was used only to assist with language refinement and manuscript organization suggestions. No content generation or substantive scientific input was provided by AI tools.

## **9. Disclosure Statement**

The author declares that there are no competing interests.

## **10. Data availability Statement**

The data that support the findings of this study, including strong motion records from the Van Nuys Hotel testbed, are openly available in PEER Center at the University of California, Berkeley: <https://apps.peer.berkeley.edu/research/peertestbeds/van%20nuys.htm>.

## **References**

- [1] Karakostas, Christos, et al. "Seismic assessment of bridges through structural health monitoring: a state-of-the-art review." *Bulletin of Earthquake Engineering* 22.3 (2024): 1309-1357.
- [2] Doebling, Scott W., et al. "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review." (1996).

- [3] Farrar, Charles R., and Keith Worden. "An introduction to structural health monitoring." *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 365.1851 (2007): 303-315.
- [4] Worden, Keith, Charles R. Farrar, and Michael D. Todd. "The Past, Present and Future of Structural Health Monitoring: An Overview of Three Ages." *Strain* (2025). Wiley. <https://doi.org/10.1111/str.12515>
- [5] Peeters, Bart, Johan Maeck, and Guido De Roeck. "Vibration-based damage detection in civil engineering: excitation sources and temperature effects." *Smart materials and Structures* 10.3 (2001): 518.
- [6] Rainieri, Carlo, and Giovanni Fabbrocino. "Operational modal analysis of civil engineering structures." Springer, New York 142 (2014): 143.
- [7] Pan, Yuxin, et al. "Model updating and seismic response of a super tall building in Shanghai." *Computers & Structures* 239 (2020): 106285.
- [8] Guyan, Robert J. "Reduction of stiffness and mass matrices." *AIAA Journal* 3.2 (1965): 380–380.
- [9] Teughels, Anne, and Guido De Roeck. "Structural damage identification of the highway bridge Z24 by FE model updating." *Journal of Sound and Vibration* 278.3 (2004): 589-610.
- [10] Mottershead, J. E., & Friswell, M. I. (1993). Model updating in structural dynamics: a survey. *Journal of Sound and Vibration*, 167(2), 347–375.
- [11] Friswell, M. I., & Mottershead, J. E. (1995). *Finite element model updating in structural dynamics*. Springer Science & Business Media.
- [12] Eshaghi, Mohammad Sadegh, Cosmin Anitescu, and Timon Rabczuk. "Methods for enabling real-time analysis in digital twins: A literature review." *Computers & Structures*, 297 (2024): 107342.
- [13] Roohi, Milad, Eric M. Hernandez, and David Rosowsky. "Reconstructing element-by-element dissipated hysteretic energy in instrumented buildings: application to the Van Nuys Hotel testbed." *Journal of Engineering Mechanics* 147.1 (2021): 04020141.

- [14] Liu, Jie, et al. "Structural damage detection and localization via an unsupervised anomaly detection method." *Reliability Engineering & System Safety*, 252 (2024): 110465.
- [15] Kaya, Y.; Safak, E. "Real-time analysis and interpretation of continuous data from structural health monitoring (SHM) systems." *Bull. Earthq. Eng.* 2014, 13, 917–934.
- [16] Sivasuriyan, Arvindan, et al. "Practical implementation of structural health monitoring in multi-story buildings." *Buildings* 11.6 (2021): 263.
- [17] Chopra, Anil K. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. 5th ed., Pearson, 2020.
- [18] Celep, Zekai. *Yapı Dinamiği [Structural Dynamics]*. 3rd ed., Birsen Publishing, 2014.
- [19] American Society of Civil Engineers. *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*. ASCE/SEI 7-16, 2017. ASCE, 2017.
- [20] Ministry of Environment and Urbanization. *Türkiye Bina Deprem Yönetmeliği 2018 [Turkish Building Earthquake Code 2018]*. 2018.
- [21] Turing, Alan M. "Rounding-off errors in matrix processes." *The Quarterly Journal of Mechanics and Applied Mathematics* 1.1 (1948): 287–308.
- [22] PEER Ground Motion Database, "RSN1176 – Kocaeli, Turkey (08/17/1999), YPT station, component YPT060", Pacific Earthquake Engineering Research Center, University of California, Berkeley, Accessed: 2024. Available at: <https://ngawest2.berkeley.edu>
- [23] Avcioğlu, Onur. *İki Parametrelili Zemin Oturan Betonarme Yapıların Doğrusal Olmayan Hesabı Ve Görsel Tabanlı Bir Bilgisayar Yazılımı*. PhD Thesis, İstanbul Teknik Üniversitesi, İstanbul, 2015.
- [24] Computers and Structures Inc. (CSI). *Etabs – Integrated Building Design Software*. Version 20, Berkeley, California, USA. Available at: <https://www.csiamerica.com/products/etabs>

- [25] Mazzoni, S., McKenna, F., Scott, M. H., and Fenves, G. L. *OpenSees Command Language Manual*. Pacific Earthquake Engineering Research Center, University of California, Berkeley, 2006. Available at: <https://opensees.berkeley.edu>
- [26] Allemang, R. J. “Investigation of Some Multiple Input/Output Frequency Response Function Experimental Modal Analysis Techniques.” Doctor of Philosophy Dissertation, University of Cincinnati, Department of Mechanical Engineering, pp. 141–214, 1980.
- [27] California Strong Motion Instrumentation Program. *Van Nuys 7-Story Hotel, CSMIP Station No. 24386*. California Division of Mines and Geology, 2001.
- [28] Krawinkler, H. (Ed.), *Van Nuys Hotel Building Testbed Report: Exercising Seismic Performance Assessment*, PEER Report 2005/11, Pacific Earthquake Engineering Research Center, University of California, Berkeley, October 2005.
- [29] Gicev, Vlado, and Mihailo D. Trifunac. “Non-linear earthquake waves in seven-storey reinforced concrete hotel.” *Soil Dynamics and Earthquake Engineering*, 29(5), 925–937, 2009. doi:10.1016/j.soildyn.2008.11.005